Technical Comments

Comment on "Proportional Navigation vs an Optimally Evading, Constant-Speed Target in Two Dimensions"

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IN a recent Note,[1] Julich and Borg present some results from a study of pursuit and evasion. The purposes of this comment are to furnish some references to related investigations, and also to take issue with a few comments and technical details.

The dynamical system described by Eqs. (1–4) in Ref. 1 is the same† as that for the "Game of Two Cars" discussed by Isaacs,[2] Meier,[3] Grantham et al.[4] and Vincent et al.[5] To be precise "The Game of Two Cars" has a fixed approach distance as a terminal target, and may use elapsed time as the cost/payoff. It is possible,[5] however, to infer behavior for the closest approach criterion from a knowledge of the game solution.

Concerning the authors' terminal target, it is of interest to note that terminating the trajectory when $\dot{r}(t_f) = 0$ does not guarantee that $r(t_f)$ is the closest approach. Discounting the observation that $r(t_f)$ may be a local maximum (in t) of $r(t)$, it is, of course, only a local extremum. It may occur that in subsequent passes the pursuer can get even closer. This behavior is seemingly requisite in differential games.

Furthermore, there is theoretically no need to resort to such an artifice. An optimal control problem with such a nonclassical payoff function [i.e., $\min_t r(t)$] can be formulated as a control problem with a state variable inequality constraint and a Mayer payoff.

For the problem of Ref. 1 this is done by adding a state variable, say c, with

$$\dot{c}(t) = 0$$

and state variable inequality

$$r(t) - c(t) \geq 0$$

The payoff to be maximized is $c(t_f)$. Cunningham[6] has analyzed games with such payoffs. Aggarwal and Leitmann[7] discuss such a control problem, including the formulation with state constraint. In fairness, the situation modeled in Ref. 1 is of practical interest since once the pursuer has flown by the evader, the former may be "blind."

Within these provisos the results presented may describe an evasive strategy that is optimal against the particular guidance law ascribed to the pursuer by the authors. As they state, the strategy is not optimal in the game theoretic sense nor does it furnish a minimax of $r(t_f)$.

Finally, from the authors' comments[1] "An interesting observation from the included results is the change of evasive strategy with $r(0)$ and $\phi(0)$. This is not apparent from the solution of the differential game formulation."

The game formulation referred to[8] is a linearized version (see, for example Ref. 9) of the geometry in Ref. 1. It is the linearization that suppresses the desired dependence. Certainly such dependence is salient in the solution of the game problem,[2–5] it is qualitatively manifested in the presence of "barriers." The reader need only consult Ref. 10 to be convinced that such barriers are numerous in the solutions of some differential games.

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† In Refs. 2–5, an evader centered coordinate system is used. Additionally, relations similar to Eq. (4) of Ref. 1 are imposed on U_p, the pursuer's turning rate.

References

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[3] Meier, L., "A New Technique for Solving Pursuit-Evasion Differential Games," Preprints of Technical Papers Tenth JACC, 1969, Scientific Press Inc, Ephrata, Pa., pp. 514–521.

[4] Grantham, W. G., Cliff, E. M. and Vincent, T. L., "On Barriers in a Problem of Collision Avoidance," *Proceedings Fourth International Conference on Systems Science*, Honolulu, 1971, pp. 136–138.

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[10] Breakwell, J. V. and Merz, A. W., "Toward a Complete Solution of the Homicidal Chauffer Game," *Proceedings of the First International Conference on Differential Games*, Amherst, Mass., 1969, pp. III-I–III-5.

Reply by Author to E. M. Cliff

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IN his comments, Cliff presents references to the "Game of Two Cars." The problem formulated in my paper was not a differential game since the strategy of the pursuer was specified to be proportional navigation. In addition time constants were included in the pursuer's guidance law and were also used for the evader in the study.

Cliff's comments regarding the termination of the trajectory are certainly correct, but as he points out, terminating the trajectory when $\dot{r} = 0$ is the case of practical interest. For example, several basic limitations prevent an aerodynamic missile from turning and making a second pass on the target.

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